MAYANG ANCHALIK COLLEGE

HOME ASSIGNMENT

2nd SEMESTER, 2nd PAPER

ECONOMICS HONOURS

PAPER : ECO-HC-2026 MATHEMATICAL

MATHODS IN ECONOMICS

NAME : MISS NITU BORA

Roll No : UA-201-340-0095

Head
Department of Economics
Mayang Anchalik College
Date....................

Question : 1. Explain with example the condition of conformability for matrix multiplication. Also show that in matrix multiplication AB is not equal to BA.

Answer : The matrices A and B are conformable for multiplication in the form AB if the number of columns of the first matrix (A) is equal to the number of rows of the second matrix (B). So that conformability condition can be expressed in terms of the dimension of the matrices A and B. If A is of dimension $m \times n$ and B is of dimension $n \times p$, then AB can be defined. But we cannot define BA as matrix product since the number of column of B is p and the number of row of A is m. So in this particular case we can multiply A by B, that is AB, but we cannot multiply B by A, that is BA. In matrix multiplication the first matrix is called 'lead' matrix and the second one is called 'lag' matrix.

From the above condition of conformability it appears that we can have the products

$\underset{(3\times 2)}{A} \cdot \underset{(2\times 2)}{B}$ and $\underset{(6\times 5)}{A} \; \underset{(5\times 3)}{B}$

but we cannot have the product

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$\underset{(2\times 2)}{B}\ \underset{(3\times 2)}{A}$ and $\underset{(5\times 3)}{B}\ \underset{(6\times 5)}{A}$

Once AB are conformable for multiplication, then what will be the dimension of the resultant matrix (c)? The dimension of the resultant matrix (c) depends on the dimensions of A and B. If AB=C, the dimension of c will be equal to the number of rows of lead matrix (A) and number of columns of lag matrix (B). So if the dimensions of A is $(m\times n)$ and the dimension of B is $n\times p$, then the dimension of the resultant matrix C will be $m\times p$, such that

$$\underset{(m\times n)}{A}\ \underset{(n\times p)}{B} = \underset{(m\times p)}{C}$$

Thus, in our examples above

$$\underset{(3\times 2)}{A}\ \underset{(2\times 2)}{B} = \underset{(3\times 2)}{C}$$

$$\underset{(6\times 5)}{A}\ \underset{(5\times 3)}{B} = \underset{(6\times 3)}{C}$$

But if both matrices are square matrices and the dimension of both the matrices are the same, then the matrices are conformable for multiplication in both the ways. In other words if the dimension of A is $(m\times m)$ and that of B is also $(m\times m)$ then we can define AB as well

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as BA and the dimension of the resultant mat-
-rix will be $(m \times m)$.

The multiplication of AB is not equal to BA. Now

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 + 5\times3 & 2\times6 + 5\times2 \\ 3\times1 + 4\times3 & 3\times6 + 4\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 22 \\ 15 & 26 \end{bmatrix}$$

and, $$BA = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2 + 6\times3 & 1\times5 + 6\times4 \\ 3\times2 + 2\times3 & 3\times5 + 2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 29 \\ 12 & 23 \end{bmatrix}$$

$$AB \neq BA$$

<u>Question : 2</u> Explain the properties of Determinants.

<u>Answer</u> : The properties of Determinats are -

① The value of a determinant does not change if the

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rows and columns are interchanged. In other words, the determinant of matrix A has the same value as the of its transpose such that $|A| = |A'|$.

For example, if $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$, then

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = 15 - 2 = 13$$

and

$$|A'| = \begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix} = 15 - 2 = 13$$

(ii) If any two rows (or any columns) are interchanged, the sign of the determinant will alter, but the numerical value will remain same.

For example if we have a determinant

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 0 & 1 \\ 3 & 2 & 2 \end{vmatrix}$$

$$|A| = 2(0-2) - 1(8-3) + 3(8-0)$$
$$= -4 - 5 + 24 = 15$$

When we interchange the first and second rows of $|A|$, then we have the determinant as

$$\begin{vmatrix} 4 & 0 & 1 \\ 2 & 1 & 3 \\ 3 & 2 & 2 \end{vmatrix} = 4(2-6) - 0 + 1(4-3)$$
$$= -16 + 1 = -15$$

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(iii) If any row (or any column) of a determinant is multiplied by a scalar (say λ), the value of the determinant will change by λ times. For example, if the first row of $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is multiplied by λ,

then $$\begin{vmatrix} \lambda a & \lambda b \\ c & d \end{vmatrix} = \lambda a.d - \lambda b.c$$

$$= \lambda(ad - bc) = \lambda \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

(iv) If one row (or one column) of a determinant is a multiple of any row (or any column), the value of the determinant will be zero. For example, for determinant

$$|A| = \begin{vmatrix} 2a & 2b \\ a & b \end{vmatrix} = 2ab - 2ab = 0$$

$$|A| = \begin{vmatrix} 4 & 6 & 2 \\ 2 & 3 & 1 \\ 5 & 1 & 2 \end{vmatrix}$$ where first row is twice the second row, then

$$|A| = 4(6-1) - 6(4-5) + 2(2-15)$$
$$= 20 + 6 - 26 = 0$$

(v) If two rows (or two columns) of a determinant are identical, the value of the determinant will be zero. For example

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$$|A| = \begin{vmatrix} 4 & 2 & 2 \\ 4 & 2 & 2 \\ 1 & 3 & 5 \end{vmatrix} = 4(10-6) - 2(20-2) + 2(12-2)$$

$$= 16 - 36 + 20 = 0$$

(vi) The addition (or subtraction) of a multiple of any row to (from) another row will not alter the value of the determinant.

Let our determinant be $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Now the first row is multiplied by λ times and then added to second row. So the determinant will be

$$\begin{vmatrix} a & b \\ \lambda a + c & \lambda b + d \end{vmatrix} = a(\lambda b + d) - b(\lambda a + c)$$

$$= \lambda ab + ad - \lambda ab - bc$$

$$= ad - bc$$

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Similarly if we subtract λ times of first row from the second row, then the determinant will be

$$\begin{vmatrix} a & b \\ c - \lambda a & d - \lambda b \end{vmatrix} = a(d - \lambda b) - b(c - \lambda a)$$

$$= ad - \lambda ab - bc + \lambda ab$$

$$= ad - bc = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

(vii) If we expand a determinant by alien co-factors, it will always give zero value. That means if we expand some row (or column) by using co-factors of some other row (or column), then the result will be zero. For example if we expand

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \\ 2 & 0 & 2 \end{vmatrix}$$

by first row elements using co-factors of third row then

$$|A| = 2\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$$

$$= 2(6-4) - 3(4-3) + 1(8-9)$$

$$= 4 - 3 - 1 = 0$$

Similarly, if we expand $|A|$ by first column using the co-factors of third column, then

$$|A| = 2\begin{vmatrix} 3 & 4 \\ 2 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$$

$$= 2(0-8) - 3(0-6) + 2(8-9)$$

$$= -16 + 18 - 2 = 0$$

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Question : 3 The demand functions of a monopoly in two different markets are given by

$$P_1 = 53 - 4Q_1$$

$$P_2 = 29 - 3Q_2$$

and the total cost function is $C = 20 + 5Q$ where P_1 and P_2 are the prices and Q_1 and Q_2 are the outputs in market 1 and market 2 respectively. Such that $Q = Q_1 + Q_2$

Solution : We are given

$$P_1 = 53 - 4Q_1$$

$$P_2 = 29 - 3Q_2$$

$$C = 20 + 5Q$$

The total revenue function of first and second markets are

$$TR_1 = P_1 \times Q_1 = 53Q_1 - 4Q_1^2$$

$$TR_2 = P_2 \times Q_2 = 29Q_2 - 3Q_2^2$$

$$C = 20 + 5Q$$

or, $$C = 20 + 5(Q_1 + Q_2)$$

or, $$C = 20 + 5Q_1 + 5Q_2$$

$$\pi = TR_1 + TR_2 - C$$

$$\pi = 53Q_1 - 4Q_1^2 + 29Q_2 - 3Q_2^2 - 20 - 5Q_1 - 5Q_2$$

First order condition for maximization

$$\frac{\partial \pi}{\partial Q_1} = 0 \text{ and } \frac{\partial \pi}{\partial Q_2} = 0$$

$\frac{\partial \pi}{\partial Q_1} = 0$ gives us

$$53 - 8Q_1 - 5 = 0$$

$$-8Q_1 = -53 + 5$$

$$+8Q_1 = +48$$

$$Q_1 = \frac{48}{8}$$

$$Q_1 = 6$$

$\frac{\partial \pi}{\partial Q_2} = 0$ gives us

$$29 - 6Q_2 - 5 = 0$$

$$-6Q_2 = -29 + 5$$

$$+6Q_2 = +24$$

$$Q_2 = \frac{24}{6}$$

$$Q_2 = 4$$

The second order condition of maximization requires that the Hessian determinants $|H_1| < 0$ and $|H_2| > 0$.

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$$|H_1| = \frac{\partial^2 \pi}{\partial Q_1^2} = -8 < 0$$

$$|H_2| = \begin{vmatrix} \frac{\partial^2 \pi}{\partial Q_1^2} & \frac{\partial^2 \pi}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 \pi}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \pi}{\partial Q_2^2} \end{vmatrix} = \begin{vmatrix} -8 & 0 \\ 0 & -6 \end{vmatrix} = 48 > 0 \text{ Satisfied}$$

Thus $Q_1 = 6$ and $Q_2 = 4$ will maximize the profit of the discriminating monopolist.

The equilibrium price are

$$P_1 = 53 - 4Q_1$$
$$= 53 - 4(6)$$
$$= 53 - 24 = 29$$

$$P_2 = 29 - 3Q_2$$
$$= 29 - 3(4)$$
$$= 29 - 12 = 17$$

The maximum profit

$$\pi = 53Q_1 - 4Q_1^2 + 29Q_2 - 3Q_2^2 - 20 - 5(Q_1 + Q_2)$$
$$= 53(6) - 4(6)^2 + 29(4) - 3(4)^2 - 20 - 5(6+4)$$
$$= 318 - 144 + 116 - 48 - 20 - 50$$
$$= 434 - 262$$
$$= 172$$

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